Bettersize
BETTER PARTICLE SIZE SOLUTIONS

A GUIDE TO PARTICLE SIZE AND SHAPE PARAMETERS BY IMAGE ANALYSIS

Bettersize Instruments Ltd.

Introduction

The particle size and shape greatly affect powder properties, which are used to evaluate product performance. Some examples are the packing density of crushed sands, the grinding effect of abrasives, the engineering properties of granular soils, etc. Various parameters are utilized to characterize the particle size and shape, which are described in this guidebook, with the intention to facilitate the understanding of these abstract concepts and provide a more comprehensive interpretation. These parameters provide insight into your samples and will be useful in new product development, product performance improvement, and quality control.

Calibration Coefficient 01

Calibration coefficient refers to the side length of a single pixel in a digital image.

Though calibration coefficient is neither a size parameter nor a shape parameter, it is of vital importance to particle size and shape analysis. Calibration coefficient can be calculated in the following order:

1. Users need to photograph an object (For example, a ruler) to obtain an image containing the complete object. The size of the object is known.

2. Users need to draw a line segment along the object. The length of the line segment can be found according to the object with known size.

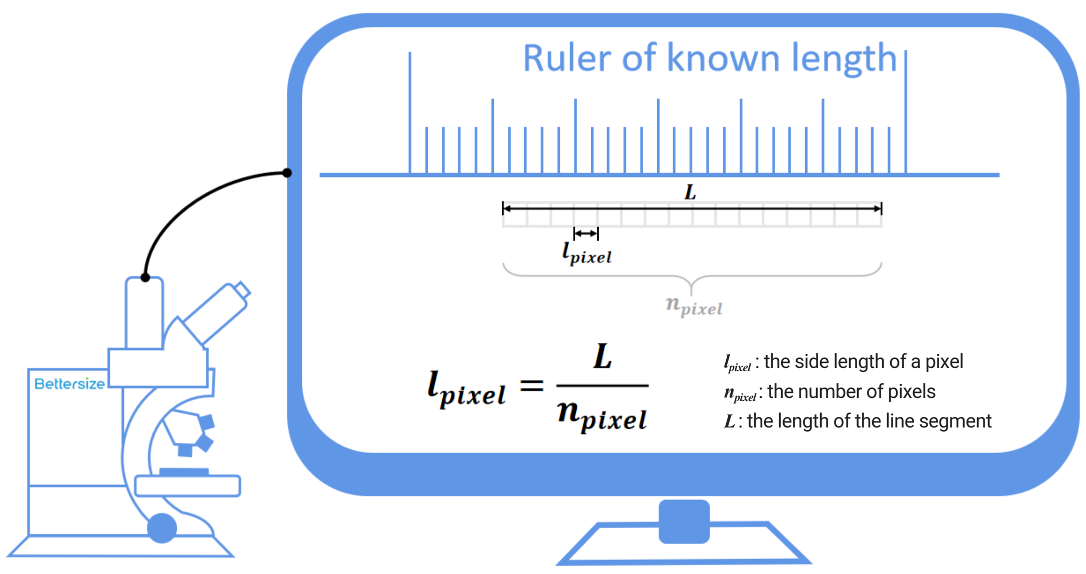
3. The line segment consists of pixels arranged one by one in a straight line. The number of pixels contained in the line segment will be automatically identified with an image analysis software.

4. Users need to input the length of the line segment and the magnification of the camera. The side length of a pixel can be therefore obtained; that is, calibration coefficient:

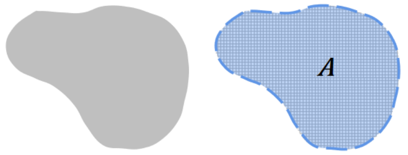
$$l_{pixel} = \frac{L}{n_{pixel}}$$

where l_{pixel} is the side length of a pixel, L is the length of the line segment, and n_{pixel} is the number of pixels contained in the line segment.

The calibration procedure shows that the obtained calibration coefficient is only applicable to the photos taken by the same camera under the same magnification. Any operations such as photo distortion, image quality compression, or camera replacement will require a new calibration.



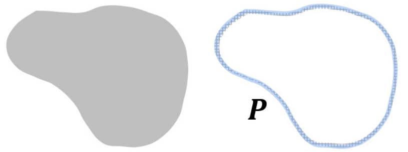
Area 02



Area refers to the projected area of a particle.

Projected area of a particle can be calculated by multiplying the number of pixels by the area of a pixel. The area of a pixel can be calculated with the help of the calibration coefficient.

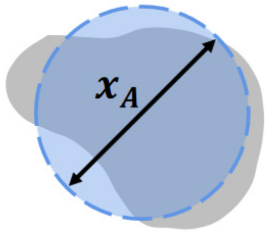
Perimeter 03

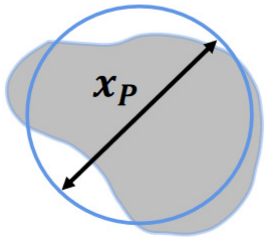


Perimeter of a particle projection refers to the total length of its projected contour.

Perimeter can be calculated through contour representation using curve fitting with the help of an image analysis software.

Area-equivalent Diameter and Perimeter-equivalent Diameter 04





Area-equivalent diameter refers to the diameter of a circle with the same area as the particle projection (Area-equivalent circle of the particle projection). **Perimeter-equivalent diameter** refers to the diameter of a circle with the same perimeter as the particle projection (Perimeter-equivalent circle of the particle projection).

The calculation formulas are shown below:

Calculation of area-equivalent diameter x_A:

$$x_A = \sqrt{\frac{4A}{\pi}}$$

where A is the projected area of the particle.

Calculation of perimeter-equivalent diameter x_P:

$$x_P = \frac{P}{\pi}$$

where P is the perimeter of the particle projection.

Circularity

05

Circularity is the ratio of area-equivalent diameter to perimeter-equivalent diameter.

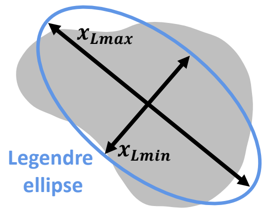
This parameter is commonly used to describe how similar the projected contour is to a circle. The circularity of a circle is 1. The closer the circularity to 1 is, the more similar the projected contour to a circle will be. The calculation of circularity is shown below:

$$Circularity = \frac{x_A}{x_P} = \sqrt{\frac{4\pi A}{P^2}}$$

where x_A is the area-equivalent diameter, x_P is the perimeter-equivalent diameter, A is the projected area of the particle, and P is the perimeter of the particle projection.

Major Axis and Minor Axis of Legendre Ellipse

06



x_{Lmax} , x_{Lmin}: lengths of the major and minor axes

Legendre ellipse is an ellipse whose geometrical moment, up to the second order, equals that of the projected area of the particle. The center of the ellipse coincides with the centroid of the projected area.

Legendre ellipse has the same inertial property as its corresponding particle projection. Generally speaking, Legendre ellipse is the abstract generalization for the inertial property of a particle projection. Therefore, the related particle shape parameters are less affected by the particle surface's edges, corners, and protrusions.

The line segment with the longest length connecting the vertices is called the **major axis**. In contrast, the **minor axis** is the line segment with the shortest length between two vertices.

Ellipse Ratio

07

Ellipse ratio refers to the ratio of the minor axis to the major axis of a Legendre ellipse.

The calculation of ellipse ratio is shown below:

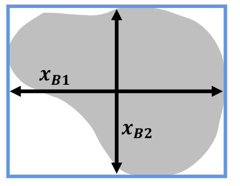
$$Ellipse\ ratio = \frac{x_{Lmin}}{x_{Lmax}}$$

where x_{Lmin} and x_{Lmax} are the lengths of the minor and major axes of the ellipse.

Ellipse ratio is used to evaluate the similarity between the projected contour and a circle. It is a relatively stable particle shape parameter, which varies little if there is a slight change in the projected contour.

Long Diameter and Short Diameter

08



Minimum bounding rectangle

x_{B1} : long diameter
x_{B2}: short diameter

The definition of **long diameter** and **short diameter** are the lengths of the long side and short side of a **minimum bounding rectangle (MBR)**.

A particle projection has an infinite number of bounding rectangles. The areas of the bounding rectangles will be calculated and compared to each other with the help of an image analysis software. The bounding rectangle with the smallest area is called the minimum bounding rectangle, whose length and width will be considered as the long diameter and the short diameter.

L/W Ratio of Minimum Bounding Rectangle 09

L/W ratio refers to the proportional relationship between the long diameter and the short diameter of a minimum bounding rectangle, i.e., the length-to-width ratio.

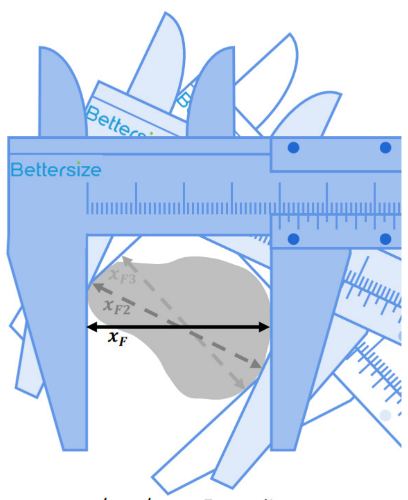
The calculation of L/W ratio is shown below:

$$L/W\ ratio = \frac{x_{B1}}{x_{B2}}$$

where x_{B1} and x_{B2} are the long and short diameters of the minimum bounding rectangle.

L/W ratio is often used to describe the degree to which a particle is stretched. This parameter can also be used in combination with aspect ratio to comprehensively evaluate the degree to which a particle is stretched or how similar its projected contour is to a circle.

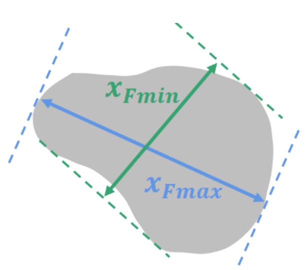
Feret Diameter 10



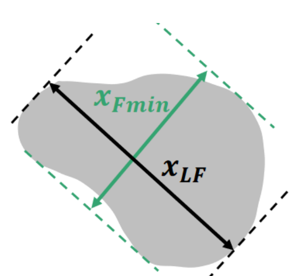
$x_F/x_{F2}/x_{F3}$: Feret diameters

Feret diameter, also known as **caliper diameter**, represents a set of diameters of a particle projection. As shown on the left, Feret diameter refers to the distance between two parallel lines tangent to the projected contour. A particle projection has an infinite number of Feret diameters.

For a particular particle projection, the **maximum Feret diameter** refers to the Feret diameter with maximum length, and the **minimum Feret diameter** is the shortest one. Besides, the Feret diameter perpendicular to the minimum Feret diameter (x_{LF}) is applied to evaluate the particle shape under certain circumstances, and it is also known as **"length"** according to the ISO 9276-6:2008.



x_{Fmax} : maximum Feret diameter
x_{Fmin}: minimum Feret diameter



x_{LF} : Feret diameter perpendicular to the minimum Feret diameter

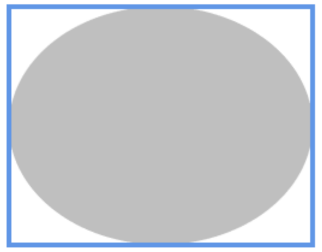
Aspect Ratio 11

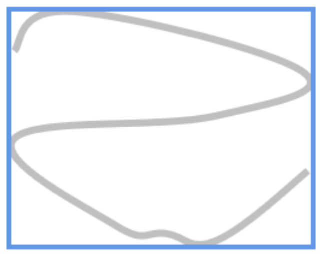
Aspect ratio refers to the proportional relationship between minimum Feret diameter and maximum Feret diameter. The calculation of aspect ratio is shown below:

$$Aspect\ ratio = \frac{x_{Fmin}}{x_{Fmax}}$$

where x_{Fmin} and x_{Fmax} are the minimum and maximum Feret diameters.

Aspect ratio is similar to L/W ratio. The combination of these two particle shape parameters is useful for evaluating the degree to which a particle is stretched or how similar its projected contour is to a circle.





Common misconception: the fiber-shaped particles cannot be easily evaluated by these two parameters due to the possible ambiguous measurement results caused by their bent or crimped shapes. In this situation, users can consider the feasibility of using these two parameters for particle shape evaluation with the help of a parameter, **straightness**, which is used to describe how flat a particle is. As shown on the left, projections of a spherical particle and a fiber-shaped particle have identical minimum bounding rectangles. However, their shapes and stretches are completely different. Users can use a parameter, **elongation**, to characterize them. For more details about elongation, please refer to Section 16.

Compactness 12

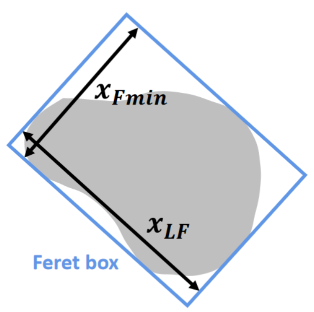
Compactness is the ratio of area-equivalent diameter to maximum Feret diameter. The calculation of compactness is shown below:

$$Compactness = \frac{x_A}{x_{Fmax}} = \frac{\sqrt{4A/\pi}}{x_{Fmax}}$$

where x_A is the area-equivalent diameter, x_{Fmax} is the maximum Feret diameter, and A is the projected area of the particle.

Compactness is commonly used to evaluate the similarity between the projected contour and a circle. The closer the compactness to 1 is, the more similar the projected contour to a circle will be.

Box Ratio 13

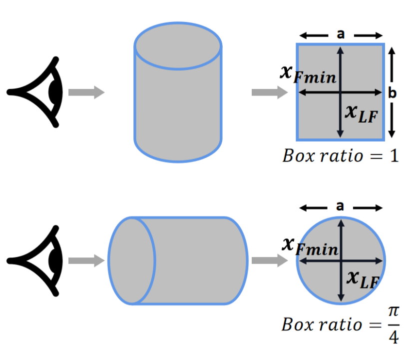


Box ratio refers to the proportional relationship between the projected area and Feret box area.

A Feret box is a rectangle whose sides consist of the minimum Feret diameter and the Feret diameter perpendicular to the minimum Feret diameter(x_{LF}).

Box ratio is commonly used to describe the degree to which the projected area of the particle fills the Feret box. The closer the box ratio to 1 is, the more similar the projected contour to its Feret box will be. The calculation of box ratio is shown below:

$$Box\,ratio = \frac{A}{A_{box}} = \frac{A}{x_{LF} \cdot x_{Fmin}}$$



where A is the projected area of the particle, A_{box} is the Feret box area, x_{Fmin} is the minimum Feret diameter, and x_{LF} is the Feret diameter perpendicular to the minimum Feret diameter.

Box ratio is a parameter that is very sensitive to projected orientation. As shown on the left, the side projection of a cylindrical particle is a rectangle whose box ratio is 1, while the box ratio of the overhead projection is only π/4.

Extent 14

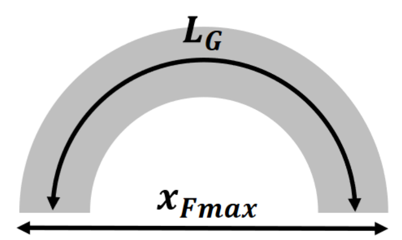
Extent, also known as **bulkiness**, is the ratio of projected area to the product of maximum Feret diameter and minimum Feret diameter.

Extent is a particle shape parameter used to evaluate the degree to which the particle is extended. The larger the extent is, the more expanded and fluffier the particle will be. The calculation of extent is shown below:

$$Extent = \frac{A}{x_{Fmax} \cdot x_{Fmin}}$$

where A is the projected area of the particle, x_{Fmax} and x_{Fmin} are the maximum and minimum Feret diameters.

Straightness 15



Very elongated particles are often bent or even curled. For a description of the degree to which a particle is bent, a parameter, **straightness**, is applied. As shown in the figure, the projection of a fiber-shaped particle is curved.

Straightness is the ratio of maximum Feret diameter to geodesic length.

When the length of a particle projection is much larger than its width, the particle length can be considered as the geodesic length of the particle. The calculation of straightness is shown below:

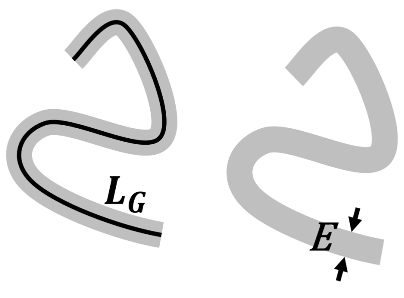
$$Straightness = \frac{x_{Fmax}}{L_G}$$

where x_{Fmax} is the maximum Feret diameter, and L_G is the geodesic length of the particle projection.

Straightness is the reciprocal of **curl**, which is also a useful particle shape parameter.

© Bettersize Instruments Ltd.

Elongation



As mentioned in the former parts, aspect ratio can be used to evaluate the degree to which a particle is stretched. For very elongated particles, a similar parameter, **elongation**, can be applied to better reflect the degree to which a fiber-shaped particle is elongated.

Elongation is the ratio of thickness to geodesic length. The calculation of elongation is shown below:

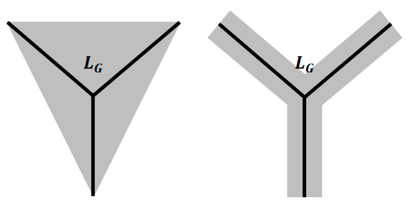
$$Elongation = \frac{E}{L_G}$$

where E is the thickness, and L_G is the geodesic length of the particle projection.

The thickness E is an average value, which can be calculated through the following formula:

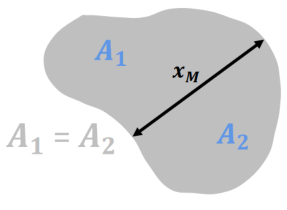
$$E = \frac{A}{L_G}$$

where A is the projected area of the particle.



Common misconception: misleading results from the elongation calculation are generated when the particles are plump and non-fibrous. As shown on the left, the geodesic for a triangular projection is Y-shaped, and its elongation is small due to the long geodesic. Before the evaluation of particle shape with elongation, users should make a preliminary evaluation of the particle shape with the help of the parameter, straightness, and then consider whether to use elongation or not. For the triangular particles, evaluation of the particle shape with aspect ratio is preferred, while evaluation using elongation is more suitable for the Y-shaped particles.

Martin Diameter



x_M : Martin diameter

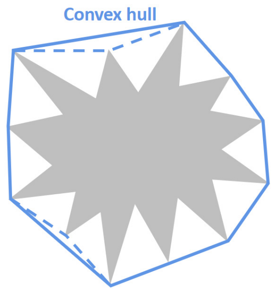
Martin diameter refers to the length of the area bisector of a particle projection.

A particle projection has an infinite number of martin diameters, of which the longest one is called the **maximum Martin diameter**, and the shortest one is called the **minimum Martin diameter**.

Convexity

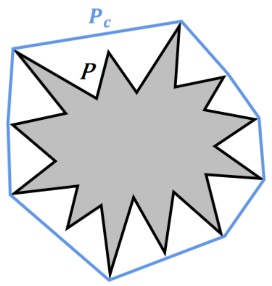
The projected contour of a particle tends to be uneven. **Convexity** is a particle shape parameter used to evaluate whether the particle surface is defective or damaged.

In geometry, a **convex hull** that corresponds to some points on a plane refers to a convex polygon, which is composed of some of the points as its vertexes. The convex hull of the points can enclose the points entirely. All interior angles of the convex hull are less than 180°.



The convex hull of a particle projection refers to a convex polygon whose vertexes are composed of some of the vertexes of the particle projection. The particle projection is enclosed in the convex hull. A convex hull for a particle projection is shown on the left. The dashed lines cannot be served as sides of the convex hull due to the interior angle being larger than 180°. The calculations of the perimeter and the area of the convex hull are the same as that of the particle projection.

Convexity refers to the ratio of the perimeter of the convex hull to the perimeter of the particle projection.



Convexity is commonly used to describe how similar the projected contour is to its convex hull. The larger the non-overlapping area inside the convex hull is, the greater the difference between the projected contour and the convex hull, and the smaller the convexity will be. The calculation of convexity is shown below:

$$Convexity = \frac{P_C}{P}$$

where P_c and P are perimeters of the convex hull and the particle projection.

Concavity 19

projected area: A

convex hull area: A_C

concave area: $A_C - A$

Concavity is a particle shape parameter used to calculate the concave area percentage of a convex hull.

The larger the concave area percentage is, the more obvious the depressions in the projected contour, and the greater the concavity will be. The calculation of concavity is shown below:

$$Concavity = \frac{A_C - A}{A_C}$$

where A_C is the area of the convex hull, and A is the projected area of the particle.

Solidity 20

Solidity is a particle shape parameter used to describe the degree to which a projected contour is concave. The calculation of solidity is shown below:

$$Solidity = \frac{A}{A_C}$$

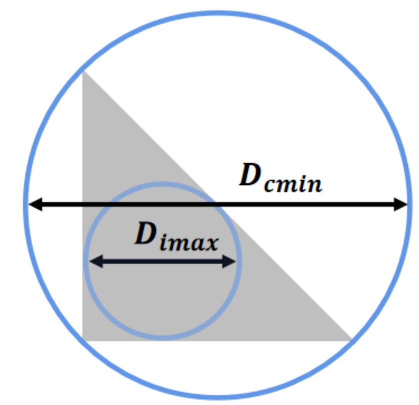
where A_C is the area of the convex hull, and A is the projected area of the particle.

The closer the solidity to 1 is, the more similar the projected contour to its convex hull, and the fewer bumps, depressions, and sharp corners on the particle surface will be.

Irregularity 21

Irregularity, also known as modification ratio, is defined as the ratio of the diameter of the maximum inscribed circle to that of the minimum circumscribed circle.

Irregularity can be used to evaluate the similarity between the projected contour and a circle. The larger and sharper the protrusions are, the larger the gap between the maximum inscribed circle and the minimum circumscribed circle of the particle projection, and the smaller the irregularity will be. The calculation of irregularity is shown below:



$$Irregularity = \frac{D_{imax}}{D_{cmin}}$$

where D_{imax} is the diameter of the maximum inscribed circle, and D_{cmin} is the diameter of the minimum circumscribed circle.

A right triangle is shown as an example in the figure. The maximum inscribed circle and the minimum circumscribed circle of the right triangle are demonstrated, and its irregularity is tan22.5° (≈ 0.414). The irregularity of the right triangle is small due to its sharp corners.

Bettersize
BETTER PARTICLE SIZE SOLUTIONS

Bettersize Instruments Ltd.

Further information can be found at

https://www.bettersizeinstruments.com

Email: info@bettersize.com

Address: No. 9, Ganquan Road, Jinquan Industrial Park, Dandong, Liaoning, China

Postcode: 118009

Tel: +86-415-6163800

Fax: +86-415-6170645

Download Our Guidebook:

Visit Our Official YouTube Channel: